## Lesson Plan



| Learni | utcomes (CPMK) |
| :---: | :---: |
| CPMK | Students are able to explain the concepts of Linear Algebra, and are skilled in applying these concepts to solve various cases related to Linear Algebra in the theoretical aspect as well as its applications in the field of Computer Science. |
| Learning outcomes of each topic (Sub-CPMK) |  |
| SubCPMK 1 | Students are able to understand the role of Linear Algebra in Computer Science as well as the basic topics of mathematics supporting Linear Algebra course. |
| Sub- CPMK <br> 2 | Students are able to understand the concept of matrices, types of matrices, and operations on matrices, and apply them in problem solving properly and correctly. |
| Sub- CPMK 3 <br> 3 | Students are able to understand the concept of the System of Linear Equations (SLE), the representation of the SLE in the form of a matrix, as well as the operations that can be performed to solve the SLE, and implement them in problem solving properly and correctly. |
| Sub- CPMK 4 | Students are able to apply Gaussian elimination and Gauss-Jordan elimination methods to solve multi-variable SLE properly and correctly. |
| SubCPMK 5 | Students are able to apply the method of calculating determinants (either combinatorial or with cofactor expansion) to compute determinants of matrices and use them in problem solving properly and correctly. |
| $\begin{gathered} \text { Sub- } \\ \text { CPMK } \end{gathered}$ $6$ | Students are able to understand the concept of the inverse of a square matrix, as well as its relationship to determinants and SLE, and apply it in problem solving properly and correctly. |
| Sub- CPMK 7 | Students are able to understand the concept of vectors in spaces $\mathrm{R}^{2,} \mathrm{R}^{3}$, and $\mathrm{R}^{\mathrm{n}}$, as well as operations related to them. |
| SubCPMK <br> 8 | Students are able to understand the concept of Euclid's vector space, general vector space, and sub-vector space, as well as related operations (addition and multiplication of scalar vectors) and apply them in solving simple problems properly and correctly. |
| Sub- CPMK 9 9 | Students are able to understand the concept of spanned sets in vector space and linear combinations between vectors in vector space, and apply the concepts to find the standard/non-standard basis and compute the dimensions of vector spaces properly and correctly. |



| Supporting: <br> - Slide Kuliah Aljabar Linier, oleh Rinaldi Munir, Institut Teknologi Bandun <br> - Slide Kuliah Aljabar Linier, oleh Dewi Sintiari |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dosen Pengampu |  | Ni Luh Dewi Sintiari, Ph.D. |  |  |  |  |  |
| Matakuliah syarat |  | Basic Mathematics |  |  |  |  |  |
| Mg <br> Ke- | Kemampuan akhir tiap tahapan belajar (Sub-CPMK) | Penilaian |  | Bantuk Pembelajaran, Learning Methods, Assignment Mahasiswa, [Estimasi Waktu] |  | Materi Pembelajaran [ Pustaka ] | Bobot Penilaian (\%) |
|  |  | Indikator | Kriteria \& Bentuk | On-site (offline) | Online |  |  |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 1 | Students are able to understand the role of Linear Algebra in Computer Science as well as the basic topics of mathematics supporting Linear Algebra course. | Accuracy in: <br> 1. explain basic mathematical concepts related to Linear Algebra courses; <br> 2. explain the urgency of understanding Linear Algebra concepts in the study of Computer Science. | Assessment Form: <br> - Non-test, oral question and answer | Learning Form: <br> Class Activities <br> [3x50'], Structured <br> Tasks [3x60'], <br> Independent <br> Learning [3x50’] <br> Learning Methods: <br> Discussion, question and answer | Media: <br> elearning.undiksha.ac .id | 1. Contract <br> 2. The role of Linear Algebra in Computer Science <br> 3. Reviewing the topics of during the semester <br> 4. Overview of basic materials related to Linear Algebra | 3\% |
| 2 | Students are able to understand the concept of matrices, types of matrices, and operations on matrices, and apply them in problem solving | Accuracy in: <br> 1. write a simple matrix correctly; <br> 2. describes the row, column, diagonal, and indices of matrix entries; | Assessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities <br> [3x50’], Structured <br> Tasks [3x60'], <br> Independent <br> Learning [3x50’] | Media: elearning.undiksha.ac .id | 1. Basics of matrices <br> 2. Matrix operations: scalar multiplication, addition, multiplication, | 5\% |


|  | properly and correctly. | 3. compute matrix operations, such as: scalar multiplication, matrix addition, matrix multiplication, matrix transpose, matrix power, and matrix polynomials; <br> 4. apply the properties of matrix operations; <br> 5. apply the concepts and properties of square matrices; <br> 6. apply the block matrix concept to solve matrix operations. |  | Learning Methods: Discussion, question and answer, assignment <br> Assignment 1: |  | transpose, exponent, matrix polynomial <br> 3. Types of square matrices: identity matrices, up/down triangular matrices, symmetric matrices, block matrices <br> 4. Properties of a square matrix: diagonal, trace, etc. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | Students are able to understand the concept of the System of Linear Equations (SLE), the representation of the SLE in the form of a matrix, as well as the operations that can be performed to solve the SLE, and implement them in problem solving properly and correctly. | Accuracy in: <br> 1. describe the components of linear equations, such as variables, coefficients, constants, the number of linear equations, and the number of variables in the system of linear equations; <br> 2. verify whether a set of values is a solution to a system of linear equations; <br> 3. formulate coefficient matrices and augmented matrices of a system of | Assessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities <br> [3x50'], Structured <br> Tasks [3x60'], <br> Independent <br> Learning [3x50'] <br> Learning Methods: <br> Discussion, question and answer, assignment <br> Assignment 2: | Media: elearning.undiksha.ac. id | 1. Fundamentals of the system of linear equations (SLE) <br> 2. SLE transformation in matrix form <br> 3. Concept of Elementary Row Operations (ERO) <br> 4. Geometric interpretation of SLE for 1, 2, or 3 variables <br> 5. Elimination and | 7\% |


|  |  | linear equations; <br> 4. identify homogeneous and non-homogeneous systems of equations, and degenerate and non-degenerate systems of equations; <br> 5. prove elementary row operations to convert a system of linear equations to another equivalent form; <br> 6. analyze the geometric interpretation of a system of linear equations with 1,2 , and 3 variables; <br> 7. apply an elimination algorithm and a substitution algorithm to solve a system of linear equations with two variables; <br> 8. explain the concept of a system of linear equations in the form of a triangular matrix and an echelon matrix form. |  |  |  | substitution methods for solving SLE <br> 6. SLE in the form of triangular matrix and echelon matrix <br> 7. The number of SLE solutions, and write down the SLE solutions <br> 8. Reduced echelon form |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | Students are able to apply Gaussian elimination and GaussJordan elimination methods to solve multivariable SLE properly and correctly. | Accuracy in: <br> 1. apply the Gaussian elimination algorithm to solve a system of linear equations with $n$ variables; <br> 2. apply the Gauss-Jordan elimination algorithm to solve a system of linear | Assessment Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: Class Activities [ $3 \times 50$ '], Structured Tasks [3x60'], Independent Learning [3x50’] <br> Learning Methods: | Media: <br> elearning.undiksha.ac. id | 1. Gaussian elimination method <br> 2. Gauss-Jordan . elimination method <br> 3. Application of SLE in Computer Science | 6\% |


|  |  | equations with $n$ variables; <br> 3. analyze the type of solution in a homogeneous system of equations (trivial solutions and non-trivial solutions); <br> 4. implementing Gauss and Gauss-Jordan elimination algorithms in programming languages; <br> 5. apply the concept of solving a system of linear equations to solve real-world related problems. |  | Discussion, question and answer, assignment <br> Assignment 3: |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | Students are able to apply the method of calculating determinants (either combinatorial or with cofactor expansion) to compute determinants of matrices and use them in problem solving properly and correctly. | Accuracy in: <br> 1. explain the concept of determinants in solving systems of linear equations; <br> 2. derive the determinant formula of a $2 x 2$ matrix through a system of linear equations; <br> 3. apply the procedure for calculating the determinant of a $3 \times 3$ matrix with the determinant formula; <br> 4. analyze the relationship of a system of linear equations 3 variables with the determinant of the coefficient matrix; <br> 5. explain the geometric interpretation of the | Assessment Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities [3x50'], Structured Tasks [3x60'], Independent Learning [3x50'] <br> Learning Methods: Discussion, question and answer, assignment <br> Assignment 4: | Media: elearning.undiksha.ac. id | 1. The concept of determinant <br> 2. The determinant formula of the $2 \times 2$, $3 \times 3$, and nxn. matrices <br> 3. Finding the SLE solution with the determinant matrix <br> 4. Geometric interpretation of matrix determinants <br> 5. Combinatorial rules for calculating determinants of matrices <br> 6. Determinant properties <br> 7. Cofactor expansion | 7\% |


|  |  | determinants of matrices of size $2 \times 2$ and $3 x 3$; <br> 6. derive the determinant formula of a matrix of size nxn; <br> 7. analyze the relationship of elementary row operations on a matrix with the determinant of the matrix; <br> 8. compute the determinant using the cofactor; <br> 9. apply Cramer's rule to solve a system of linear equations; <br> 10. explain the relationship of the block matrix and the determinant of the matrix. |  |  |  | 8. Cramer's Rules <br> 9. Determinant of block matrix |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | Students are able to understand the concept of the inverse of a square matrix, as well as its relationship to determinants and SLE, and apply it in problem solving properly and correctly. | Accuracy in: <br> 1. explain the concept of the inverse matrix and its relationship to the determinant of the matrix; <br> 2. formulate the procedure for calculating the inverse of a 2x2 . matrix; <br> 3. formulate the procedure for calculating the inverse of a 2x2 . matrix; <br> 4. explain the concepts of orthogonality and orthonomality; <br> 5. prove the inverse properties of matrices; | Assessment Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities <br> [3x50'], Structured <br> Tasks [3x60’], <br> Independent <br> Learning [3x50’] <br> Learning Methods: <br> Discussion, question and answer, assignment <br> Assignment 5: | Media: <br> elearning.undiksha.ac. id | 1. Calculation of the inverse of $2 \times 2$ and $3 x 3$. square matrices <br> 2. Inverse matrix nxn with adjoin <br> 3. Orthogonal matrix <br> 4. Orthonomalities in orthogonal matrices <br> 5. Properties of the inverse matrix <br> 6. Matrix inverse relation with Gauss elimination method, Gauss-Jordan elimination. | 7\% |


|  |  | 6. apply the Gaussian elimination algorithm to compute the inverse of the matrix; <br> 7. apply the Gauss-Jordan elimination algorithm to compute the inverse of the matrix; <br> 8. apply the concept of inverse matrix to solve a system of linear equations (homogeneous and nonhomogeneous). |  |  |  | 7. Rank matrix |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | Students are able to understand the concept of vectors in spaces $\mathrm{R}^{2,}$ $\mathrm{R}^{3}$, and $\mathrm{R}^{\mathrm{n}}$, as well as operations related to them. | Accuracy in: <br> 1. explain the concept of vectors in Linear Algebra, algebraically and geometrically; <br> 2. explain the concept of vectors in Linear Algebra, algebraically and geometrically; <br> 3. explain the concept of spatial vectors (in $\mathrm{R}^{3}$ ) using unit vectors $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$; <br> 4. compute the dot product between vectors; <br> 5. compute vector norms, distances between two vectors, angles between vectors and vector projections; | Assessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> - Class Activities [3x50'] <br> - Assignment mandiri [3x50'], Structured Tasks [3x60'], Independent Learning [3x50'] <br> Learning Methods: Discussion, question and answer, assignment <br> Assignment 6: | Media: <br> elearning.undiksha.ac .id | 1. The concept of vectors in spaces $R^{2}$ and $\mathrm{R}^{3}$ <br> 2. Operations between vectors on $R^{2}$ and $\mathrm{R}^{3}$ <br> 3. Properties of vectors in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$ <br> 4. Geometric interpretation of vector operations in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$ <br> 5. Vector in space $R^{n}$ | 7\% |


|  |  | 6. compute the cross product between two 3 dimensional vectors; <br> 7. apply the properties of dot multiplication and cross multiplication in calculations; <br> 8. formulate the cofactor expansion to compute the cross multiplication of 3dimensional vectors; <br> 9. to interpret geometrically the cross product in space $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 |  |  |  |  |  |  | 10\% |
| 9 | Students are able to understand the concept of Euclid's vector space, general vector space, and sub-vector space, as well as related operations (addition and multiplication of scalar vectors) and apply them in solving simple problems properly and correctly. | Accuracy in: <br> 1. explain the concept of Euclid's vector space with dimension n ; <br> 2. perform vector addition operations, vector scalar multiplication, and linear combinations between vectors in space $\mathrm{R}^{\mathrm{n}}$; <br> 3. deriving the properties of vector operations on Rn ; <br> 4. interpret geometrically the linear combination between vectors in $\mathrm{R}^{2}$; <br> 5. explain the concept of general vector space through the axioms of | Assessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities [3x50'], Structured Tasks [3x60'], Independent Learning [3x50'] <br> Learning Methods: Discussion, question and answer, assignment <br> Assignment 7: | Media: elearning.undiksha.ac. id | 1. Euclid vector space <br> 2. Common vector space <br> 3. Vector subspace | 7\% |


|  |  | vector space; <br> 6. connect the concept of Euclid's vector space and general vector space; <br> 7. prove whether a set of vectors forms a vector space; <br> 8. explain the concept of vector sub-space; <br> 9. prove the vector sub-space theorem; <br> 10. prove whether a set of vectors is a sub-vector space of a given vector space. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | Students are able to understand the concept of spanned sets in vector space and linear combinations between vectors in vector space, and apply the concepts to find the standard/nonstandard basis and compute the dimensions of vector spaces properly and correctly. | Accuracy in: <br> 1. explain the concept of spanning set in a vector space through linear combinations; <br> 2. identify linearly independent and non-linearly independent vectors in the space $\mathrm{R}^{\mathrm{n}}$; <br> 3. explain the concept of basis in vector space $R n$ and general vector space; <br> 4. explain the difference between standard and nonstandard basis of a vector space; <br> 5. explain the concept of vector space dimensions and | Assessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities [3x50'], Structured Tasks [3x60'], Independent Learning [3x50’] <br> Learning Methods: Discussion, question and answer, assignment <br> Assignment 8: | Media: elearning.undiksha.ac. id | 1. Combination liner <br> 2. Linear independence <br> 3. Base vector space Rn and general vector space <br> 4. Standard and nonstandard base <br> 5. Dimensions of vector space and sub-vector space | 7\% |


|  |  | their relation to vector space bases; <br> 6. find the dimensions of a vector space. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | Students are able to perform transformation between bases in a vectors space, and relate it to the column, row, and null spaces. | Accuracy in: <br> 1. derive the transformation matrix from one basis to another in the same vector space; <br> 2. explain the concept of column space; <br> 3. explain the concept of row space; <br> 4. explain the concept of null space; <br> 5. find the rank and nullity of a vector space. | Assessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities <br> [3x50'], Structured <br> Tasks [3x60'], <br> Independent <br> Learning [3x50’] <br> Learning Methods: <br> Discussion, question and answer, assignment <br> Assignment 9: | Media: elearning.undiksha.ac. id | 1. Vector space base replacement <br> 2. Matrix column space <br> 3. Matrix row space <br> 4. Null space | 5\% |
| 12 | Students are able to understand the concept of linear transformation and solve related problems properly and correctly. | Accuracy in: <br> 1. explain the concept of transformation and transformation matrices; <br> 2. explain the concept of linear transformation; <br> 3. find the standard transformation matrix of a linear transformation; | Assessment <br> Form: <br> - Tanya-jawab lisan <br> - Assignment membuat video | Learning Form: <br> Class Activities <br> [3x50'] <br> Learning Methods: <br> Discussion, question and answer, assignment | Media: elearning.undiksha.ac. id | 1. Transformation concept <br> 2. Zero transformation and identity operator <br> 3. Properties of linear transformation <br> 4. The concept of linear | 5\% |


|  |  | 4. derive the standard matrix for the reflection vector at $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$; <br> 5. derive the standard matrix for vector projection in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$; <br> 6. derive the standard matrix for the rotation of the vectors in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$; <br> 7. derive the standard matrix for the dilation and contraction vectors in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$; <br> 8. derive the standard matrix for vector expansion and compression in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$; <br> 9. derive the standard matrix for the shear vector transformation in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$; <br> 10. determine the result of vector transformation composition. |  | Assignment 10: |  | transformation <br> 5. Standard matrix of linear transformation <br> 6. Transformations in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$ : reflection, projection, rotation, dilation, expansion, shear <br> 7. Properties of transformation matrices |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | Students are able to understand the concepts of eigenvalues, eigenvectors, eigenspaces, and matrix diagonalizations, and solve related problems properly and correctly. | Accuracy in: <br> 1. explain the concept of eigenvalues and eigenvectors; <br> 2. compute the eigenvalues of the matrix; <br> 3. compute the eigenvector matrix; <br> 4. compute the basis of the | Bentuk <br> PeniAssessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment laian: <br> - Non-test, | Learning Form: <br> Class Activities <br> [3x50'], Structured <br> Tasks [3x60'], <br> Independent <br> Learning [3x50'] <br> Learning Methods: <br> Discussion, | Media: elearning.undiksha.ac .id | 1. The concept of eigenvalues <br> 2. The concept of eigenvectors <br> 3. Base eigenspace <br> 4. Diagonalization <br> 5. Application of eigenvectors | 7\% |


|  |  | eigenspace matrix; <br> 5. explain the concept of matrix diagonalization; <br> 6. explain the properties of the matrix that is maintained in the diagonalization operation; <br> 7. analyze whether a matrix can be diagonalized; <br> 8. find a matrix that can be used to diagonalize a matrix; <br> 9. diagonalize the matrix. | oral question and answer <br> - Assignment | question and answer, assignment <br> Assignment 11: |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14 | Students are able to understand the concept of inner product and related operations, inner product space, GramSchmidt procedure, and solve related problems properly and correctly. | Accuracy in: <br> 1. explain the concept of product in; <br> 2. compute the inner product of two vectors; <br> 3. compute the angle between two vectors with the principle of the inner product; <br> 4. compute the distance between two vectors; <br> 5. investigate the orthogonality of two vectors with an inner product; <br> 6. explain the concepts of orthogonal sets and orthonormal sets; <br> 7. compute the QR decomposition of a matrix; | Assessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities [3x50'], Structured Tasks [3x60'], Independent Learning [3x50’] <br> Learning Methods: Discussion, question and answer, assignment <br> Assignment 12: | Media: elearning.undiksha.ac .id | 1. Inner product space <br> 2. Angles and orthogonalities in the inner product space <br> 3. Gram-Schmidt <br> 4. QR Decomposition <br> 5. Least squares problem <br> 6. Application of deep product space | 7\% |


|  |  | 8. explain the concept of the least squares problem; <br> 9. determine the least squares solution of a system of linear equations. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15 | Students are able to understand the concept of matrix decomposition, such as QR decomposition, LU decomposition, and singular value decomposition, and solve related problems properly and correctly. | Accuracy in: <br> 1. explain the concept of matrix decomposition; <br> 2. determine the singular value of a matrix; <br> 3. apply singular value decomposition algorithm to a matrix; <br> 4. explain the application of singular value decomposition of a matrix. | Assessment <br> Form: <br> - Non-test, oral question and answer <br> - Assignment | Learning Form: <br> Class Activities <br> [3x50'], Structured <br> Tasks [3x60'], <br> Independent <br> Learning [3x50’] <br> Learning Methods: <br> Discussion, question and answer, assignment <br> Assignment 13: | Media: elearning.undiksha.ac .id | 1. Types of matrix decomposition <br> 2. Singular value decomposition <br> 3. An example of applying singular value decomposition | 5\% |
| 16 | Final Evaluation |  |  |  |  |  | 15\% |

